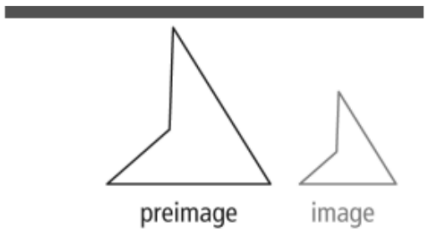
Chapter 3 Transformations

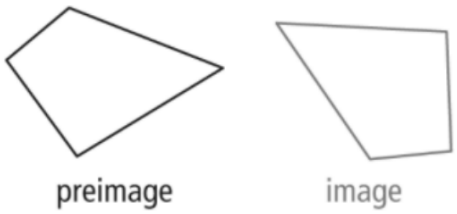
A rigid motion is a transformation that preserves length and angle measure. Is the transformation a rigid motion? Explain.

SOLUTION

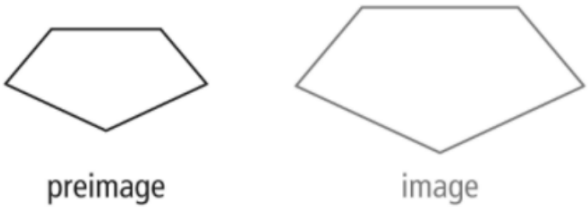


Try It!

1. a. Is the transformation a rigid motion? Explain.



1. b. Is the transformation a rigid motion? Explain.

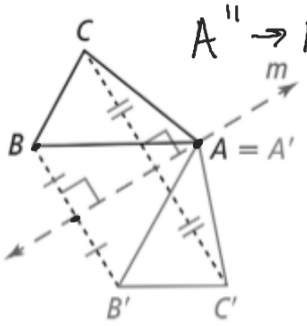


Reflections

$A' \rightarrow$ A Prime

$A'' \rightarrow$ A Double Prime

A reflection is a transformation that reflects each point in the preimage across a line of reflection.



A reflection has these properties:

- If a point A is on line m, then the point and its image are the same point (that is, $A' = A$).
- If a point B is not on line m, line m is the perpendicular bisector of $\overline{BB'}$.

The reflection of $\triangle ABC$ across line m can be written as $R_m(\triangle ABC) = \triangle A'B'C'$.

A reflection is a rigid motion so length and angle measures are preserved.

Quadrilateral $FGHJ$ has coordinates $F(0,\ 3)$, $G(2,\ 4)$, $H(4,\ 2)$, $J(-2,\ 0)$.

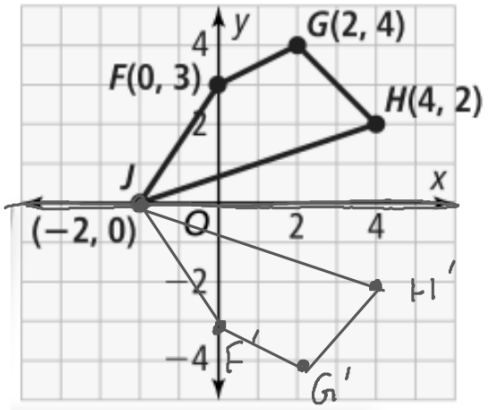
A. Graph and label $FGHJ$ and $R_{x\text{-axis}}(FGHJ)$. What is a general rule for reflecting a point across the x-axis?

$F(0,3) \rightarrow F'(0,-3)$

$G(2,4) \rightarrow G'(2,-4)$

$H(4,2) \rightarrow H'(4,-2)$

$J(-2,0) \rightarrow J'(-2,0)$



Quadrilateral $FGHJ$ has coordinates $F(0,\ 3)$, $G(2,\ 4)$, $H(4,\ 2)$, $J(-2,\ 0)$.

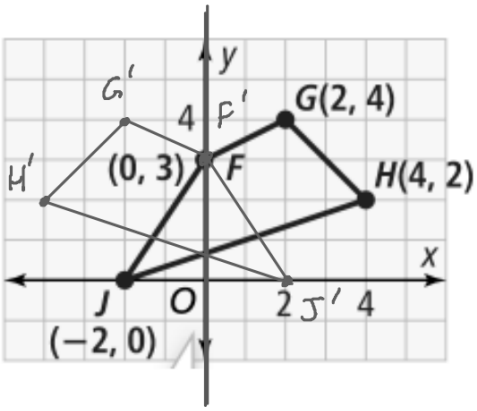
B. Graph and label $FGHJ$ and $R_{y\text{-axis}}(FGHJ)$. What is a general rule for reflecting a point across the *y*-axis?

$F(0,3) \rightarrow F'(0,3)$

$G(2,4) \rightarrow G'(-2,4)$

$H(4,2) \rightarrow H'(-4,2)$

$J(-2,0) \rightarrow J'(2,0)$



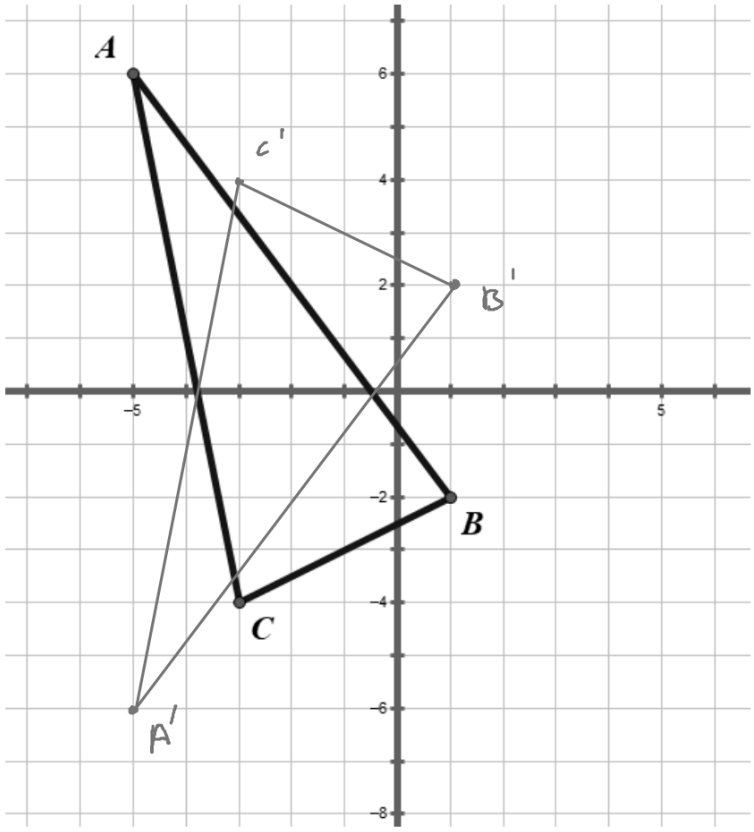
3. Triangle ABC has vertices $A(-5,\ 6)$, $B(1,\ -2)$, and $C(-3,\ -4)$. What are the coordinates of the vertices of $\triangle A'B'C'$ for each reflection?

a. $R_{x\text{-axis}}$ $(x, y) \rightarrow (x, -y)$

$A(-5, 6) \rightarrow A'(-5, -6)$

$B(1, -2) \rightarrow B'(1, 2)$

$C(-3, -4) \rightarrow C'(-3, 4)$



3. Triangle ABC has vertices $A(-5,\ 6)$, $B(1,\ -2)$, and $C(-3,\ -4)$. What are the coordinates of the vertices of $\triangle A'B'C'$ for each reflection?

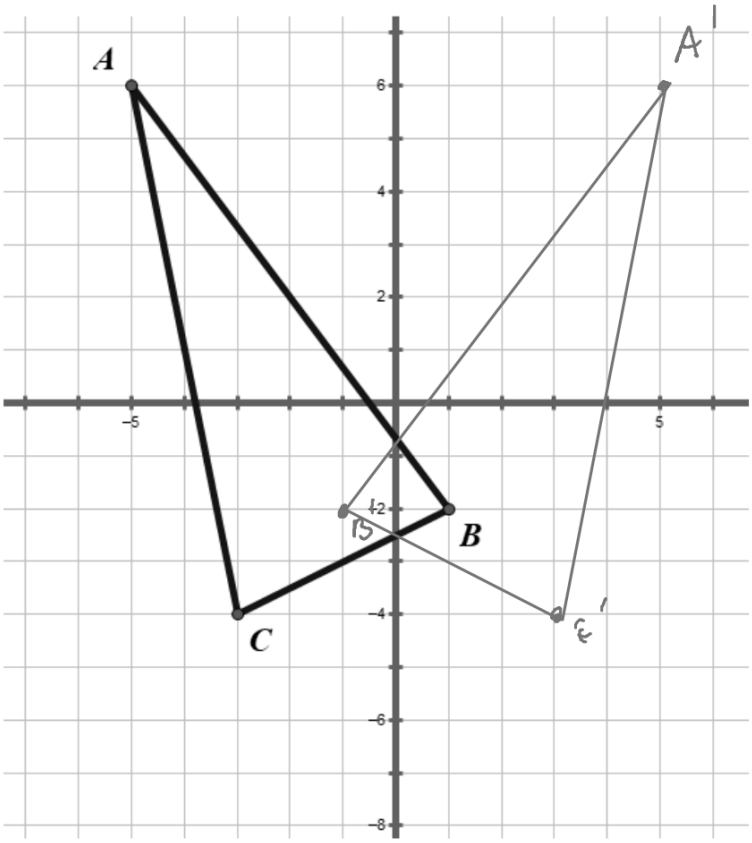
$(x,y) \rightarrow (-x,y)$

$R_{y\text{-axis}}$

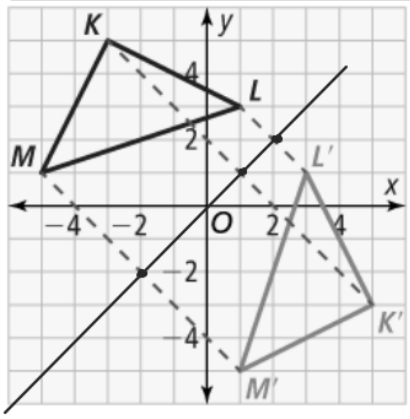
$A(-5,6) \rightarrow A'(5,6)$

$B(1,-2) \rightarrow B'(-1,-2)$

$C(-3,-4) \rightarrow C'(3,-4)$



What reflection rule maps $\triangle KLM$ to its image?



Step 1
Write the coordinates of the preimage and the image.

$K\,(-3,\ 5)$	$L\,(1,\ 3)$	$M\,(-5,\ 1)$
$K'\,(5,\ -3)$	$L'\,(3,\ 1)$	$M'\,(1,\ -5)$

Step # 2 Find midpt $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

KK'

$\left(\frac{-3+5}{2} \quad \frac{5+(-3)}{2}\right)$

$(1,1)$

LL'

$\left(\frac{1+3}{2}, \frac{3+1}{2}\right)$

$(2,2)$

MM'

$\left(\frac{-5+1}{2}, \frac{1+(-5)}{2}\right)$

$(-2,-2)$

Step #3

Choose 2 points from above and write the equation of line

$(2,2) \qquad (-2,-2)$

Find Slope $= \frac{y_2-y_1}{x_2-x_1} = \frac{-2-2}{-2-2} = \frac{-4}{-4} = 1$

$y-2 = 1(x-2)$

$y-2 = x-2$

$+2 \quad +2$

$y = x$

4. What is a reflection rule that maps each triangle to its image?

a. $C(3, 8), D(5, 12), E(4, 6)$ and $C'(-8, -3), D'(-12, -5), E'(-6, -4)$

CC'

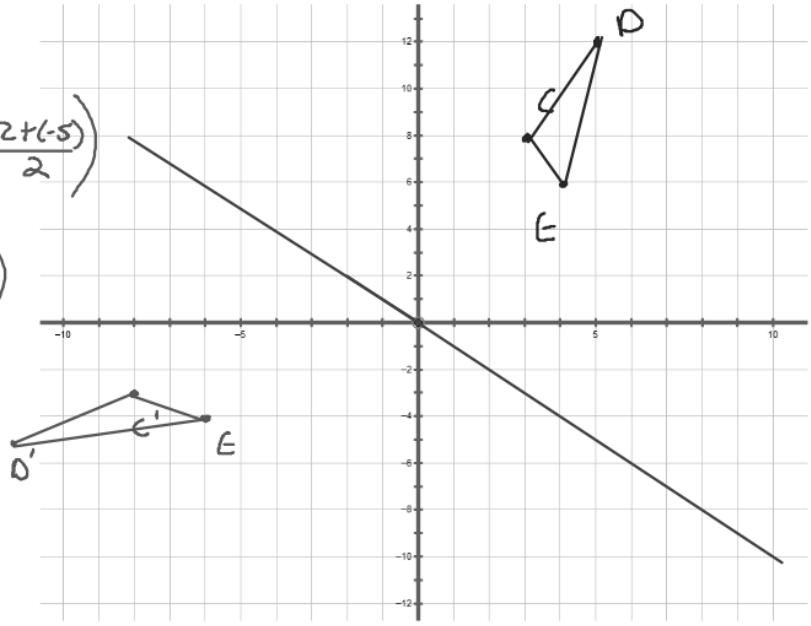
$\left(\frac{3+(-8)}{2}, \frac{8+(-3)}{2}\right)$

$\left(-\frac{5}{2}, \frac{5}{2}\right)$

DD'

$\left(\frac{5+(-12)}{2}, \frac{12+(-5)}{2}\right)$

$\left(-\frac{7}{2}, \frac{7}{2}\right)$



$$m = \frac{\frac{7}{2} - \frac{5}{2}}{-\frac{7}{2} - \left(-\frac{5}{2}\right)} = \frac{1}{-1}$$

$$= -1$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{7}{2} = -1\left(x + \frac{7}{2}\right)$$

$$y - \frac{7}{2} = -x - \frac{7}{2}$$

$$y = -x$$

. What is a reflection rule that maps each triangle to its image?

$F(7, 6), G(0, -4), H(-5, 0)$ and $F'(-5, 6), G'(2, -4), H'(7, 0)$

$F(7,6) \rightarrow F'$

$X = 1$

FF'

$\left(\frac{7-5}{2}, \frac{6+6}{2}\right)$

$(1, 6)$

GG'

$\left(\frac{0+2}{2}, \frac{-4+-4}{2}\right)$

$(1, -4)$

$m = \frac{-4-6}{1-1} = \frac{-10}{0}$ = undefined

$X = 1$